Section 5.1 (Day 2) – Practice with Identities!

A few more identities to know

Cofunction Identities:

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x \quad \sec\left(\frac{\pi}{2}-x\right)=\csc x \quad \tan\left(\frac{\pi}{2}-x\right)=\cot x$$
$$\cos\left(\frac{\pi}{2}-x\right)=\sin x \quad \csc\left(\frac{\pi}{2}-x\right)=\sec x \quad \cot\left(\frac{\pi}{2}-x\right)=\tan x$$

Odd and Even Identities:

$$\sin(-x)=-\sin x \quad \cos(-x)=\cos x \quad \tan(-x)=-\tan x$$
$$\csc(-x)=-\csc x \quad \sec(-x)=\sec x \quad \cot(-x)=-\cot x$$

Let's do some more examples WITH THE NEW IDENTITIES:

Simplify (to one trig function):

a) $\dfrac{\sin(-x)}{\cos(-x)} = \dfrac{-\sin x}{\cos x}$

$= -\tan x$

b) $\dfrac{\sin\left(\frac{\pi}{2}-z\right)}{\cos\left(\frac{\pi}{2}-z\right)} = \dfrac{\cos z}{\sin z} = \cot z$

c) $\dfrac{1-\sin^2(-x)}{\csc^2(-x)-1} = \dfrac{\cos^2(-x)}{\cot^2(-x)} = \dfrac{(\cos x)^2}{(-\cot x)^2}$

$= \dfrac{\cos^2 x}{\cot^2 x}$

$= +\cos^2 x \tan^2 x$

$= +\cos^2 x\left(\dfrac{\sin^2 x}{\cos^2 x}\right) = +\sin^2 x$

d) $\cos\left(\frac{\pi}{2}-x\right)\sec x = \sin x\left(\dfrac{1}{\cos x}\right)$

$= \tan x$

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Back to the other kind:

a) $\cos^2\beta(1+\tan^2\beta)$

$\cos^2\beta + \cos^2\beta\tan^2\beta$

$\cos^2\beta + \cos^2\beta\frac{\sin^2\beta}{\cos^2\beta}$

$= \cos^2\beta + \sin^2\beta$

$= 1$

b) $(2\csc x+2)(2\csc x-2)$

$4\csc^2 x - 4 = 4(\csc^2 x - 1) = 4\cot^2 x$

Sometimes it is not one trig function at the end – it is just a simpler combined expression:

a) *Add* $\frac{1}{1+\cos x}+\frac{1}{1-\cos x}$

$= \frac{1-\cos x + 1+\cos x}{1-\cos^2 x}$

$= \frac{2}{\sin^2 x} = 2\csc^2 x$

b) *Subtract* $\tan x - \frac{\sec^2 x}{\tan x}$

$\frac{\tan^2 x - \sec^2 x}{\tan x} = \frac{-1}{\tan x}$

$= -\cot x$

c) *factor:* $\tan^4 x+2\tan^2 x+1$

$x^4 + 2x^2 + 1$

$(x^2+1)(x^2+1)$

$(\tan^2 x + 1)(\tan^2 x + 1)$

$= \sec^2 x \quad \sec^2 x$

$= \sec^4 x$

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Write ***without*** a fraction:

a) $\frac{\sin^2 y}{1-\cos y}$

$= \frac{1-\cos^2 y}{1-\cos y}$

$= \frac{(1+\cos y)(1-\cos y)}{1-\cos y}$

$= 1+\cos y$

b) $\frac{\cos^2 y}{1-\sin y}$

$= \frac{1-\sin^2 y}{1-\sin y}$

$= \frac{(1+\sin y)(1-\sin y)}{1-\sin y}$

$= 1+\sin y$

HW: p.379 # 19, 20, 33-41, 48, 49, 53, 56, 58, 63